

1

Sets – Learning Outcomes

- Use set notation and terminology.
- List elements of a finite set.
- Describe the rule that defines a set.
- Describe and recognise equality of sets.
- Perform intersection, union.
- Investigate commutativity for intersection and union.
- Illustrate sets using Venn diagrams.
- Use and discuss universal set, null set, and subsets.
- Perform set difference.
- Investigate commutativity for set difference.
- Perform set complement.

1

2

Use Set Notation and Terminology

- A **set** is a collection of objects, e.g.
 - furniture in a room,
 - players on a team,
 - students in this class,
 - numbers 1-10.
- Write down three more examples of sets.

- Sets are usually labelled with a capital letter.

2

3

€ Use Set Notation and Terminology



- Objects in a set are called **elements** of that set.
- Notation for this is \in , e.g.
 - armchair \in furniture in my living room,
 - Jamie Heaslip \in Irish rugby team,
 - Lia \in students in Mr. Lawless' maths class,
 - 4 \in numbers 1-10
- Write one element of each of your sets using proper notation.

3

4 **∉** Use Set Notation and Terminology

- Objects that are not in a set are not elements of that set.
- Notation for this is \notin , e.g.
 - wardrobe \notin living room furniture,
 - Scarlett Johanssen \notin Irish rugby team,
 - Mr. Lawless \notin students in Mr. Lawless' maths class,
 - 20 \notin numbers 1-10.
- Write one object that is not an element of each of your sets using proper notation.

4

5 **∈** Use Set Notation and Terminology

∉

- If $B = \{w, x, y, z\}$, fill in the blanks using one of these symbols: \in , \notin
 - d ___ B
 - w ___ B
- $C = \{m, n, o, p, 5, 6, 7, 8\}$. Fill in the blanks with \in or \notin .
 - m ___ C
 - v ___ C
 - 3 ___ C
 - 5 ___ C

5

6 **{ }** List Elements of a Finite Set

- We write the elements of a set between a pair of curly brackets $\{ \}$, e.g.
 - Furniture in my living room = {couch, armchair, table, dining chair, end table, lamp}
 - Irish rugby team = {Jamie Heaslip, Johnny Sexton, Rory Best, Andrew Trimble, ...}
 - Students in Mr. Lawless' maths class = {Oskar, Alice, Finn, Arthur, ...}
 - Numbers 1-10 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
- Write the elements of your three sets using proper notation.

6

7 } List Elements of a Finite Set

1. If A is the set of odd numbers between 1 and 20, write the elements of A using set notation.
2. If B is the set of letters in the word Wicklow, write the elements of B using set notation.
3. Write the set of cities in Ireland.
4. Write the set of prime numbers less than 30.

7


8 # Use Set Notation and Terminology

- The number of elements in a set is called the set's **cardinal number** or **cardinality**.
- The symbol # before a set means its cardinal number, e.g.
 - #furniture in my living room = 16.
 - #Irish rugby team = 33.
 - #students in Mr. Lawless' maths class = 20.
 - #numbers 1-10 = 10
- Write the cardinal number for each of your sets using proper notation.

8

9 # Use Set Notation and Terminology

1. Write down a set which has a cardinal number of 4.
2. Using your set of prime numbers less than 30, find the number of elements in that set.
3. If the box below is a set and its contents are elements, what is the cardinality of the box?



by Stephan Kulla - public domain

9

10

Describe the Rule that Defines a Set

- Give the rule that defines each of the following sets by describing their elements.
 - $A = \{1, 2, 3, 4, 5\}$
 - $B = \{9, 12, 15, 18\}$
 - $C = \{1, 4, 9, 16, 25\}$
 - $D = \{2, 3, 5, 7, 11, 13\}$
 - $E = \{k, l, n, y\}$. If the elements of E come from the word "Kilkenny", describe the rule that picks these elements.

10

11

Describe and Recognise Equality of Sets

- Two sets are **equal** if they have exactly the same elements.
 - e.g. $M = \{\text{banana, orange, apple}\}$ and $N = \{\text{apple, banana, orange}\}$. $M = N$.
 - e.g. $A = \{2, 4, 6, 8\}$ and $B = \{8, 4, 6, 2\}$. $A = B$.

11

12

Describe and Recognise Equality of Sets

- $A = \{3, 6, 9\}$ and $B = \{\text{multiples of 3 less than 10}\}$. Are A and B equal?
- $C = \{3, 5, 15\}$ and $D = \{\text{factors of 15}\}$. Are C and D equal?
- $R = \{l, t, w\}$ and $S = \{\text{consonants in the word "trowel"}\}$. Are R and S equal?
- $W = \{\text{vowels in the word "following"}\}$ and $X = \{\text{vowels in the word "join"}\}$. List the elements of A and B, and state whether or not they are equal.

12

13 **∩** Perform Intersection of Sets

- Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$
- The **intersection** of two sets is the list of elements in *both* sets.
- Notation for this is $A \cap B$.
- e.g. $A \cap B = \{3, 4\}$

13

14 **∪** Perform Union of Sets

- Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$
- The **union** of two sets is the list of all elements from *either* set.
- Notation for this is $A \cup B$.
- e.g. $A \cup B = \{1, 2, 3, 4, 5, 6\}$

14

15 **∩** Perform Intersection and Union

- $E = \{4, 6, 9\}$ and $F = \{4, 5\}$. Write the intersection and union of E and F using set notation.
- $M = \{1, 2, 3, a, b, c\}$ and $N = \{2, 4, 6, b, d, f\}$. Write the intersection and union of M and N using set notation.
- First year maths teachers = {Ms. Cadden, Ms. Crowe, Mr. Doel, Mr. Hipwell, Mr. Lawless, Mr. Macken, Ms. O'Hanlon, Ms. O'Reilly}, First year science teachers = {Mr. Fahey, Ms. King, Mr. Macken, Mr. Ryan, Ms. Teehan}. Write the intersection and union for the sets of maths and science teachers using set notation.

15

16

Investigate Commutativity

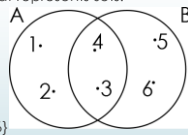
- **Commutativity** is when changing the order of things doesn't change the result.
- e.g. $3 + 4 = 4 + 3$
- e.g. $2 \times 5 = 5 \times 2$
- If $A = \{w, x, y, z\}$ and $B = \{u, v, w, x\}$, investigate if:
 - $A \cap B = B \cap A$
 - $A \cup B = B \cup A$

16

17

Illustrate Sets using Venn Diagrams

- A **Venn diagram** is a picture that represents sets.
- Venn diagrams have:
 - a circle for each set.
 - a label on each circle.
 - a dot for each element.
 - overlaps show intersection.
- e.g. $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$



17

18

Illustrate Sets using Venn Diagrams

1. Draw a Venn diagram to represent $C = \{2, 4, 6, 8, 10\}$ and $D = \{5, 6, 7, 8\}$.
2. Draw a Venn diagram to represent $M = \{a, b, c, d, e\}$ and $N = \{a, e, i, o, u\}$.
3. Draw a Venn diagram to represent $W = \{1, 3, 5, 7, 9\}$ and $X = \{2, 6, 10, 11\}$.
4. Draw a Venn diagram to represent $Y = \{\text{even numbers less than } 10\}$ and $Z = \{\text{prime numbers less than } 10\}$.

18

19

Illustrate Sets using Venn Diagrams

- Based on the Venn diagram below, write out the sets:
- A
- B
- $A \cap B$
- $A \cup B$

19

20

Illustrate Sets using Venn Diagrams

- Some Venn diagrams do not show the elements of a set, instead showing the *cardinality* of each set.
- e.g. the diagram below shows how many people in a survey like each type of chocolate.

- How many people like both types of chocolate?
- How many people like only one type of chocolate?
- How many people like milk chocolate?

20

21

Use and Discuss Subsets

- Subsets** are sets entirely contained inside another set.
- Notation for this is \subset , e.g.
 - $A = \{3, 6\}$, $B = \{3, 6, 9\}$, then $A \subset B$.
- If $C = \{1, 2, 3\}$, its subsets are:
 - $\{\}$ (a.k.a. the **null set**, \emptyset)
 - $\{1\}$, $\{2\}$, $\{3\}$
 - $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$
 - $\{1, 2, 3\}$ (C itself, a.k.a. the **improper subset**)

21

22

☞ Use and Discuss Subsets

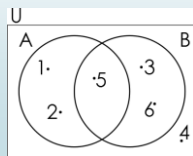
- Sets that are not entirely contained within another set are not subsets.
 - Notation for this is $\not\subset$, e.g.
 - $A = \{3, 6\}$, $B = \{3, 5, 9\}$, then $A \not\subset B$.
1. If $K = \{1, 4, 9\}$, list all the subsets of K .
 2. Given $F = \{a, c, e, g, 2, 4, 6, 8\}$, fill in the blanks using one of these symbols: \subset , $\not\subset$.
 - $\{a, e, g\}$ ___ F
 - $\{c, e, 6, 10\}$ ___ F
 - K ___ F

22

23

Use and Discuss Universal Sets

- The **universal set**, U (not union, u) is the set that contains every element.
- In practice it contains only elements of interest for the problem at hand.
- It is shown as a box on a Venn diagram.

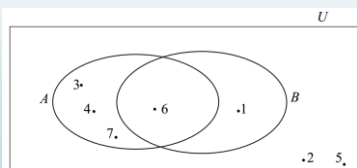


23

24

Use and Discuss Universal Sets

1. For the Venn diagram below, write out:
 - U, A, B
 - $A \cap B$
 - $A \cup B$

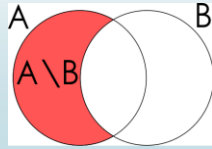


24

25

Perform Set Difference

- The **difference** of A and B is the set of elements in A that are not in B.
- We use the symbol \setminus for set difference.
- e.g. $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 7, 9\}$
- $A \setminus B = \{1, 2, 3, 4, 5\} \setminus \{1, 3, 5, 7, 9\} = \{2, 4\}$



25

26

Perform Set Difference

- Given the sets $A = \{2, 4, 6, 8, 10, 12, 14\}$, $B = \{3, 6, 9, 12, 15\}$ and $C = \{5, 6, 7, 8, 9, 10, 11\}$.
- a) Draw Venn diagrams showing:
 - i. Sets A and B
 - ii. Sets B and C
 - iii. Sets A and C
- b) Using your Venn diagrams, find:

i. $A \setminus B$	iv. $(B \cap C) \setminus A$
ii. $B \setminus C$	v. $(B \cup C) \setminus C$
iii. $C \setminus A$	vi. $(A \cap B) \setminus C$

26

27

Investigate Commutativity

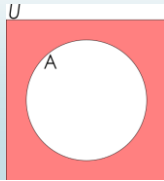
- Draw a Venn diagram for each of the following pairs of sets, and find $A \setminus B$ and $B \setminus A$.
- 1. $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8, 10\}$
- 2. $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 6, 9, 12, 15\}$
- 3. $A = \{2, 3, 5, 7, 11\}$, $B = \{1, 4, 9, 16, 25\}$
 - Does $A \setminus B = B \setminus A$?
- Write down five elements of the set $C = \mathbb{Z} \setminus \mathbb{N}$

27

28

Perform Complement

- The **complement** of A is the set of elements that are not in A.
- We use the symbol ' or C for set complement.
- e.g. $A = \{1, 2, 4, 8, 10\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- $A' = A^C = \{3, 5, 6, 7, 9\}$



28

29

Perform Complement

- Draw a Venn diagram for each of the following questions and use it to find A'
1. $A = \{1, 2, 3, 4, 5\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 2. $A = \{1, 3, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 3. $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$, $B = \{1, 4, 9, 16, 25\}$, $U = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$

29
