

Sets – Learning Outcomes

- Use set notation and terminology.
- List elements of a finite set.
- Describe the rule that defines a set.
- Describe and recognise equality of sets.
- Perform intersection, union.
- Investigate commutativity for intersection and union.
- Illustrate sets using Venn diagrams.
- Use and discuss universal set, null set, and subsets.
- Perform set difference.
- Investigate commutativity for set difference.
- Perform set complement.

Use Set Notation and Terminology

- ▶ A **set** is a collection of objects, e.g.
 - ▶ furniture in a room,
 - ▶ players on a team,
 - ▶ students in this class,
 - ▶ numbers 1-10.
- ▶ Write down three more examples of sets.
- ▶ Sets are usually labelled with a capital letter.

€ Use Set Notation and Terminology

- Objects in a set are called **elements** of that set.
- Notation for this is €, e.g.
 - armchair € furniture in my living room,
 - Jamie Heaslip € Irish rugby team,
 - Lia € students in Mr. Lawless' maths class,
 - 4 € numbers 1-10
- Write one element of each of your sets using proper notation.





Use Set Notation and Terminology

- ▶ Objects that are not in a set are not elements of that set.
- ▶ Notation for this is \notin , e.g.
 - ▶ wardrobe \notin living room furniture,
 - ▶ Scarlett Johanssen \notin Irish rugby team,
 - ▶ Mr. Lawless \notin students in Mr. Lawless' maths class,
 - ▶ 20 \notin numbers 1-10.
- ▶ Write one object that is not an element of each of your sets using proper notation.



Use Set Notation and Terminology



1. If $B = \{w, x, y, z\}$, fill in the blanks using one of these symbols: \in , \notin

► d ___ B

► w ___ B

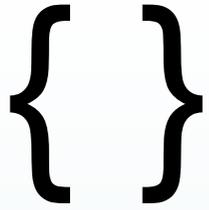
2. $C = \{m, n, o, p, 5, 6, 7, 8\}$. Fill in the blanks with \in or \notin .

► m ___ C

► v ___ C

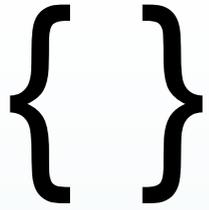
► 3 ___ C

► 5 ___ C



List Elements of a Finite Set

- ▶ We write the elements of a set between a pair of curly brackets $\{ \}$, e.g.
 - ▶ Furniture in my living room = {couch, armchair, table, dining chair, end table, lamp}
 - ▶ Irish rugby team = {Jamie Heaslip, Johnny Sexton, Rory Best, Andrew Trimble, ... }
 - ▶ Students in Mr. Lawless' maths class = {Oskar, Alice, Finn, Arthur, ...}
 - ▶ Numbers 1-10 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
- ▶ Write the elements of your three sets using proper notation.



List Elements of a Finite Set

1. If A is the set of odd numbers between 1 and 20, write the elements of A using set notation.
2. If B is the set of letters in the word Wicklow, write the elements of B using set notation.
3. Write the set of cities in Ireland.
4. Write the set of prime numbers less than 30.

Use Set Notation and Terminology

- ▶ The number of elements in a set is called the set's **cardinal number** or **cardinality**.
- ▶ The symbol # before a set means its cardinal number, e.g.
 - ▶ #furniture in my living room = 16,
 - ▶ #Irish rugby team = 33,
 - ▶ #students in Mr. Lawless' maths class = 20,
 - ▶ #numbers 1-10 = 10
- ▶ Write the cardinal number for each of your sets using proper notation.

Use Set Notation and Terminology

1. Write down a set which has a cardinal number of 4.
2. Using your set of prime numbers less than 30, find the number of elements in that set.
3. If the box below is a set and its contents are elements, what is the cardinality of the box?



Describe the Rule that Defines a Set

- ▶ Give the rule that defines each of the following sets by describing their elements.
 1. $A = \{1, 2, 3, 4, 5\}$
 2. $B = \{9, 12, 15, 18\}$
 3. $C = \{1, 4, 9, 16, 25\}$
 4. $D = \{2, 3, 5, 7, 11, 13\}$
 5. $E = \{k, l, n, y\}$. If the elements of E come from the word “Kilkenny”, describe the rule that picks these elements.

== Describe and Recognise Equality of Sets

- ▶ Two sets are **equal** if they have exactly the same elements.
- ▶ e.g. $M = \{\text{banana, orange, apple}\}$ and $N = \{\text{apple, banana, orange}\}$. $M = N$.
- ▶ e.g. $A = \{2, 4, 6, 8\}$ and $B = \{8, 4, 6, 2\}$. $A = B$.

Describe and Recognise Equality of Sets

1. $A = \{3, 6, 9\}$ and $B = \{\text{multiples of 3 less than 10}\}$. Are A and B equal?
2. $C = \{3, 5, 15\}$ and $D = \{\text{factors of 15}\}$. Are C and D equal?
3. $R = \{l, t, w\}$ and $S = \{\text{consonants in the word "trowel"}\}$. Are R and S equal?
4. $W = \{\text{vowels in the word "following"}\}$ and $X = \{\text{vowels in the word "join"}\}$. List the elements of A and B, and state whether or not they are equal.



Perform Intersection of Sets

- ▶ Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$
- ▶ The **intersection** of two sets is the list of elements in *both* sets.
- ▶ Notation for this is $A \cap B$.
- ▶ e.g. $A \cap B = \{3, 4\}$

U

Perform Union of Sets

- ▶ Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$
- ▶ The **union** of two sets is the list of all elements from *either* set.
- ▶ Notation for this is $A \cup B$.
- ▶ e.g. $A \cup B = \{1, 2, 3, 4, 5, 6\}$



Perform Intersection and Union

1. $E = \{4, 6, 9\}$ and $F = \{4, 5\}$. Write the intersection and union of E and F using set notation.
2. $M = \{1, 2, 3, a, b, c\}$ and $N = \{2, 4, 6, b, d, f\}$. Write the intersection and union of M and N using set notation.
3. First year maths teachers = $\{\text{Ms. Cadden, Ms. Crowe, Mr. Doel, Mr. Hipwell, Mr. Lawless, Mr. Macken, Ms. O'Hanlon, Ms. O'Reilly}\}$. First year science teachers = $\{\text{Mr. Fahey, Ms. King, Mr. Macken, Mr. Ryan, Ms. Teehan}\}$. Write the intersection and union for the sets of maths and science teachers using set notation.

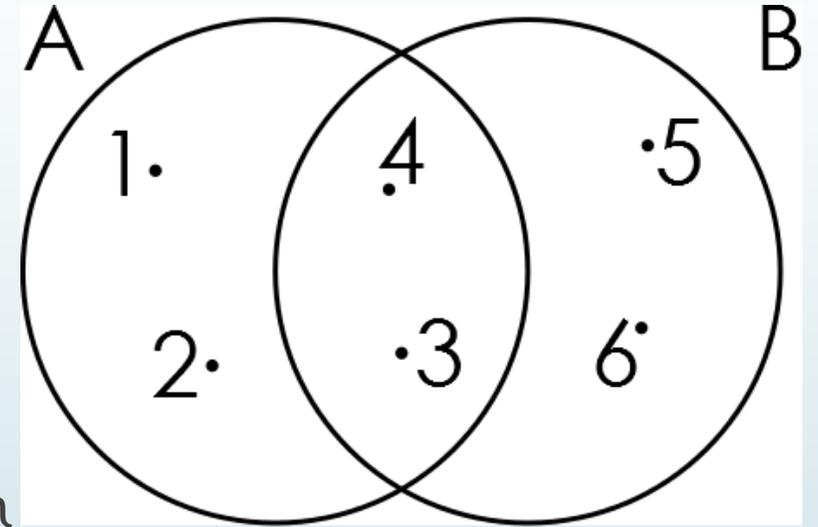
Investigate Commutativity

- ▶ **Commutativity** is when changing the order of things doesn't change the result.
- ▶ e.g. $3 + 4 = 4 + 3$
- ▶ e.g. $2 \times 5 = 5 \times 2$

- ▶ If $A = \{w, x, y, z\}$ and $B = \{u, v, w, x\}$, investigate if:
 - ▶ $A \cap B = B \cap A$
 - ▶ $A \cup B = B \cup A$

Illustrate Sets using Venn Diagrams

- ▶ A **Venn diagram** is a picture that represents sets.
- ▶ Venn diagrams have:
 - ▶ a circle for each set.
 - ▶ a label on each circle.
 - ▶ a dot for each element.
 - ▶ overlaps show intersection.
- ▶ e.g. $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$

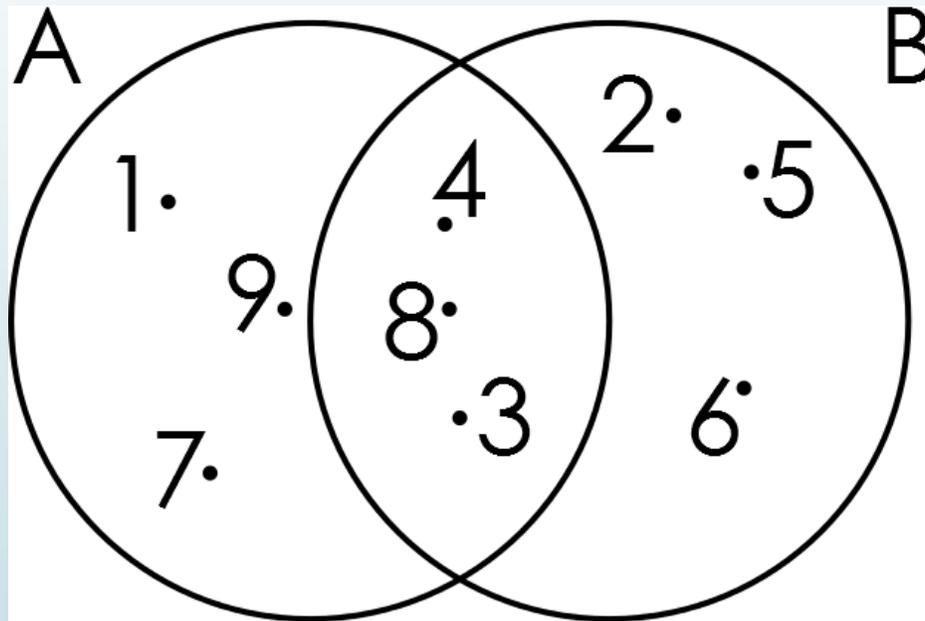


Illustrate Sets using Venn Diagrams

1. Draw a Venn diagram to represent $C = \{2, 4, 6, 8, 10\}$ and $D = \{5, 6, 7, 8\}$.
2. Draw a Venn diagram to represent $M = \{a, b, c, d, e\}$ and $N = \{a, e, i, o, u\}$.
3. Draw a Venn diagram to represent $W = \{1, 3, 5, 7, 9\}$ and $X = \{2, 6, 10, 11\}$.
4. Draw a Venn diagram to represent $Y = \{\text{even numbers less than } 10\}$ and $Z = \{\text{prime numbers less than } 10\}$.

Illustrate Sets using Venn Diagrams

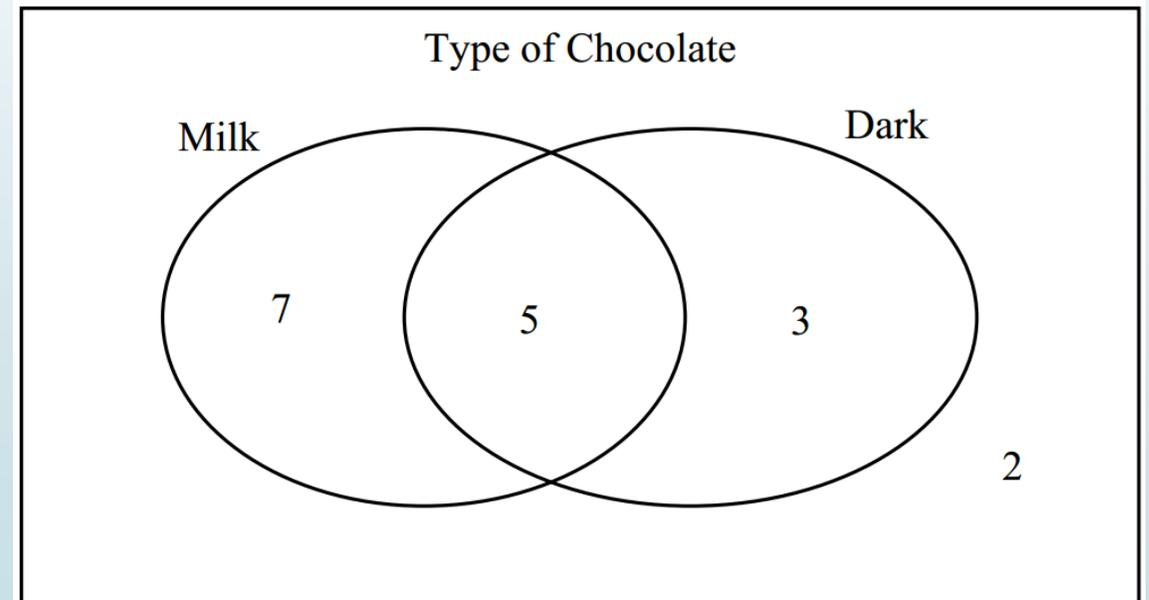
- Based on the Venn diagram below, write out the sets:
- A
- B
- $A \cap B$
- $A \cup B$



Illustrate Sets using Venn Diagrams

- ▶ Some Venn diagrams do not show the elements of a set, instead showing the *cardinality* of each set.
- ▶ e.g. the diagram below shows how many people in a survey like each type of chocolate.

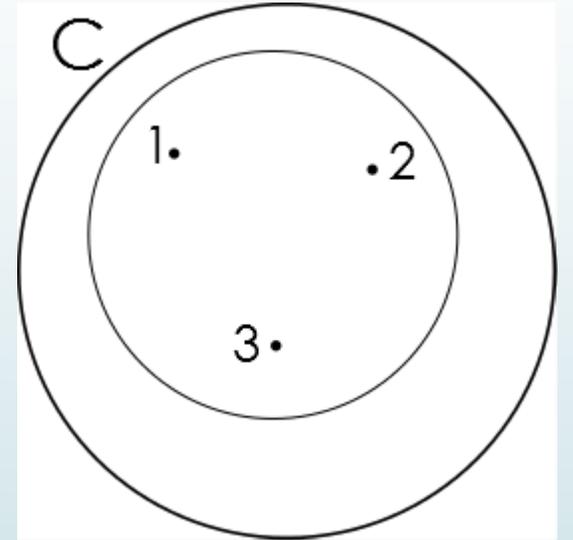
1. How many people like both types of chocolate?
2. How many people like only one type of chocolate?
3. How many people like milk chocolate?





Use and Discuss Subsets

- ▶ **Subsets** are sets entirely contained inside another set.
- ▶ Notation for this is \subset , e.g.
 - ▶ $A = \{3, 6\}$, $B = \{3, 6, 9\}$, then $A \subset B$.
- ▶ If $C = \{1, 2, 3\}$, its subsets are:
 - ▶ $\{\}$ (a.k.a. the **null set**, \emptyset)
 - ▶ $\{1\}$, $\{2\}$, $\{3\}$
 - ▶ $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$
 - ▶ $\{1, 2, 3\}$ (C itself, a.k.a. the **improper** subset)



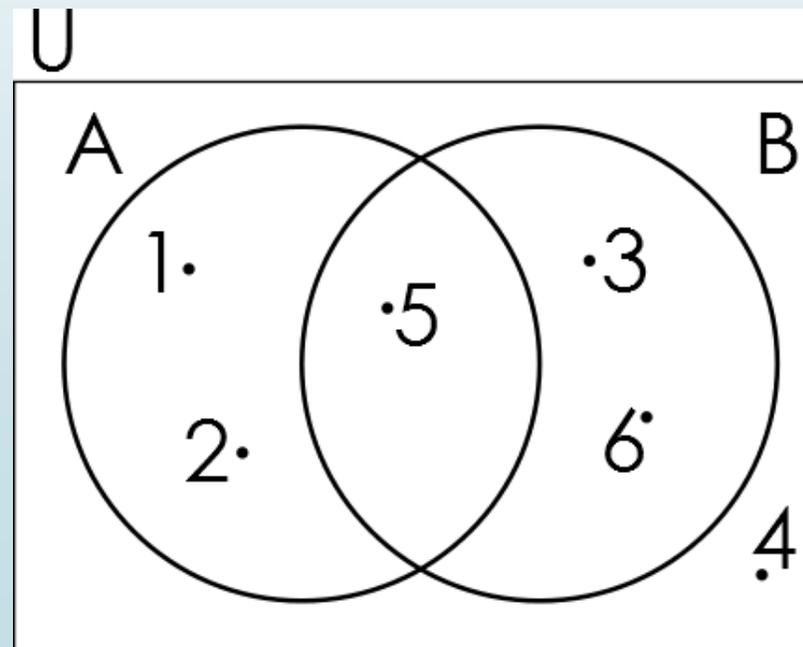


Use and Discuss Subsets

- ▶ Sets that are not entirely contained within another set are not subsets.
- ▶ Notation for this is $\not\subset$, e.g.
 - ▶ $A = \{3, 6\}$, $B = \{3, 5, 9\}$, then $A \not\subset B$.
- 1. If $K = \{1, 4, 9\}$, list all the subsets of K .
- 2. Given $F = \{a, c, e, g, 2, 4, 6, 8\}$, fill in the blanks using one of these symbols: \subset , $\not\subset$.
 - ▶ $\{a, e, g\}$ ___ F
 - ▶ $\{c, e, 6, 10\}$ ___ F
 - ▶ K ___ F

Use and Discuss Universal Sets

- ▶ The **universal set**, U (not union, u) is the set that contains every element.
- ▶ In practice it contains only elements of interest for the problem at hand.
- ▶ It is shown as a box on a Venn diagram.



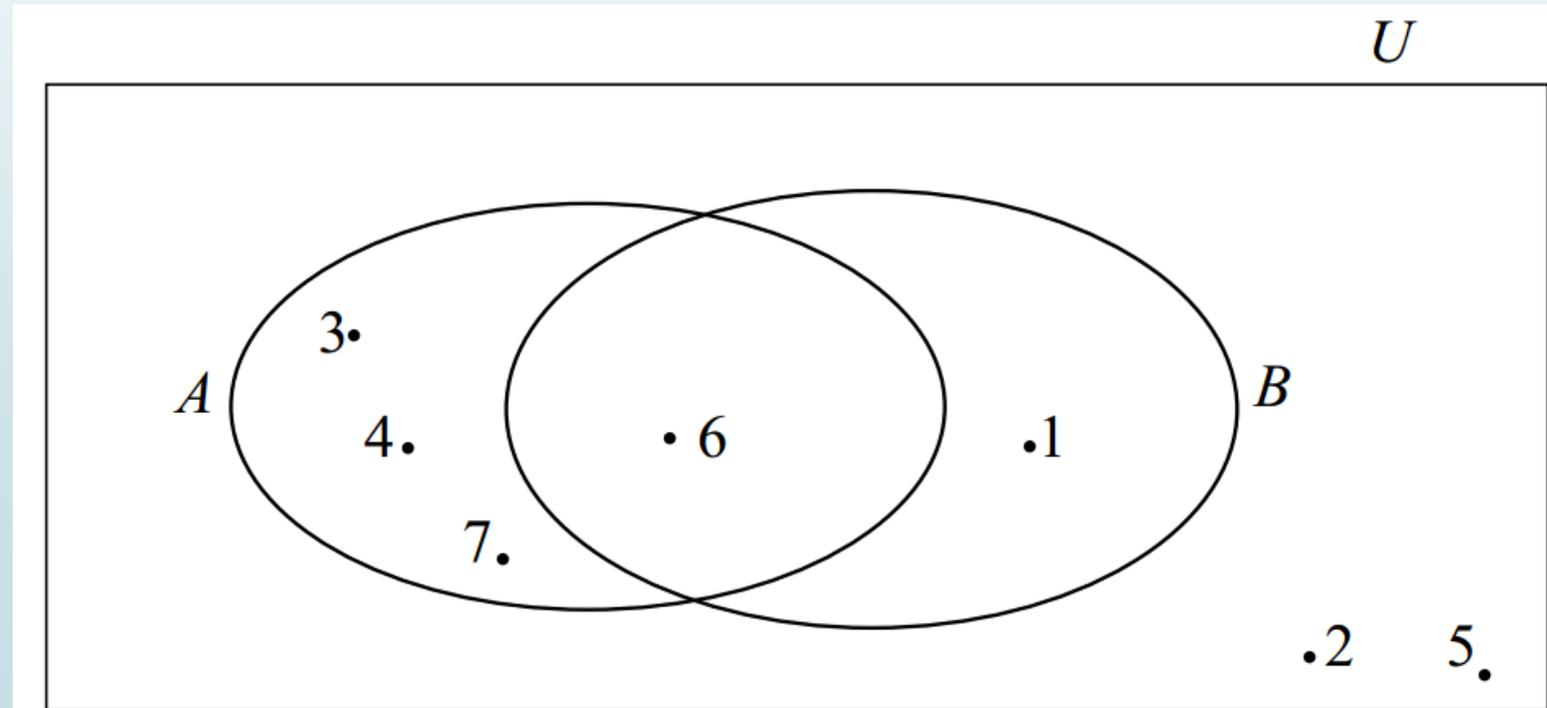
Use and Discuss Universal Sets

1. For the Venn diagram below, write out:

► U, A, B

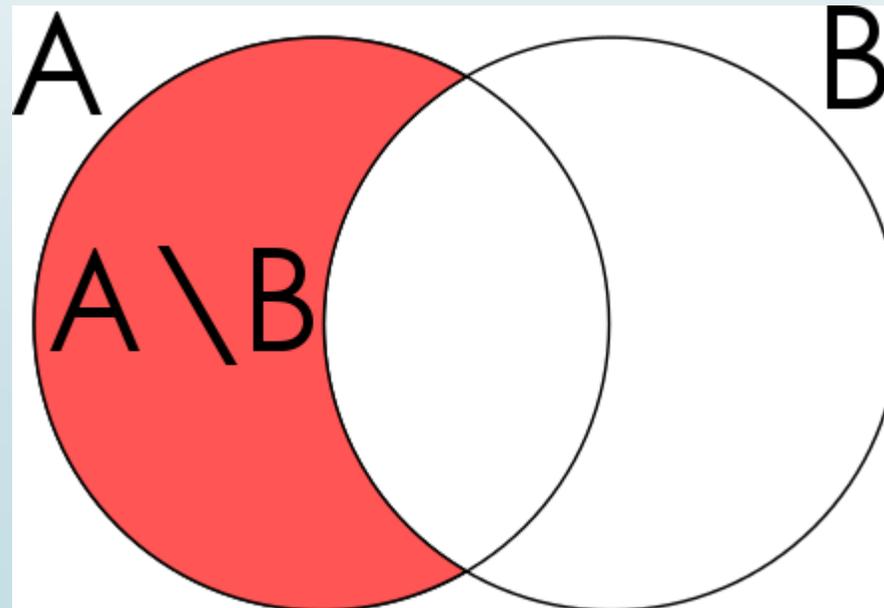
► $A \cap B$

► $A \cup B$



Perform Set Difference

- ▶ The **difference** of A and B is the set of elements in A that are not in B.
- ▶ We use the symbol \setminus for set difference.
- ▶ e.g. $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 7, 9\}$
- ▶ $A \setminus B = \{1, 2, 3, 4, 5\} \setminus \{1, 3, 5, 7, 9\} = \{2, 4\}$



Perform Set Difference

► Given the sets $A = \{2, 4, 6, 8, 10, 12, 14\}$, $B = \{3, 6, 9, 12, 15\}$ and $C = \{5, 6, 7, 8, 9, 10, 11\}$.

a) Draw Venn diagrams showing:

- i. Sets A and B
- ii. Sets B and C
- iii. Sets A and C

b) Using your Venn diagrams, find:

- | | |
|----------------------|------------------------------|
| i. $A \setminus B$ | iv. $(B \cap C) \setminus A$ |
| ii. $B \setminus C$ | v. $(B \cup C) \setminus C$ |
| iii. $C \setminus A$ | vi. $(A \cap B) \setminus C$ |

Investigate Commutativity

► Draw a Venn diagram for each of the following pairs of sets, and find $A \setminus B$ and $B \setminus A$.

1. $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8, 10\}$

2. $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 6, 9, 12, 15\}$

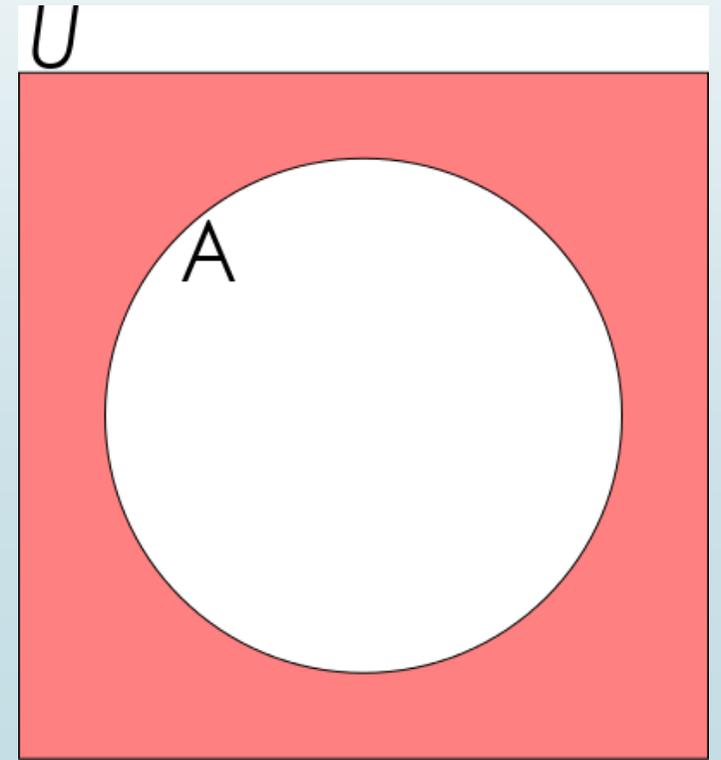
3. $A = \{2, 3, 5, 7, 11\}$, $B = \{1, 4, 9, 16, 25\}$

► Does $A \setminus B = B \setminus A$?

► Write down five elements of the set $C = \mathbb{Z} \setminus \mathbb{N}$

Perform Complement

- ▶ The **complement** of A is the set of elements that are not in A .
- ▶ We use the symbol $'$ or C for set complement.
- ▶ e.g. $A = \{1, 2, 4, 8, 10\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- ▶ $A' = A^C = \{3, 5, 6, 7, 9\}$



Perform Complement

► Draw a Venn diagram for each of the following questions and use it to find A'

1. $A = \{1, 2, 3, 4, 5\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

2. $A = \{1, 3, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

3. $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$, $B = \{1, 4, 9, 16, 25\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$