Decimals – Outcomes

- Represent addition, subtraction, multiplication, and division in $\mathbb{Q}$ using number lines and decomposition.

- Perform addition, subtraction, multiplication, and division in $\mathbb{Q}$.

- Convert between fractions and decimals.

- Present numerical answers to a specified degree of accuracy (e.g. to two decimal places or three significant figures).

Represent ℚ Using Decomposition

- Most ancient cultures used different symbols for different-sized numbers.

- e.g. Romans used "I" to represent 1, "V" to represent 5, "X" to represent 10.

- e.g. Babylonians used "Ĭ" to represent 1 and "⦊" to represent 10.

- Combinations of these would give each other number.

- e.g. Romans would write 23 as "XXIII"

- e.g. Babylonians would write 23 as "⦊⦊ĬĬĬ"

Represent ℚ Using Decomposition

- Today, we use the decimal system.

- There are ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

- Each symbol has a different value depending on where it is in the number:

- e.g. 297.5 means:

  - 2 hundreds

  - 9 tens

  - 7 units

  - 5 tenths

- We can decompose it by writing as the sum of these:

- $200 + 90 + 7 + \dfrac{5}{10}$

Represent ℚ Using Decomposition
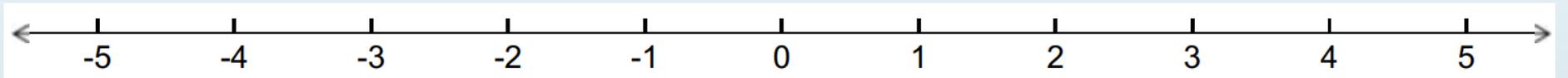
➡ Decompose each of the following:

a) 3.3

b) 6.5

c) 1111

d) 21.7

e) 208.36

f) 1 000 001

g) 8765.4321
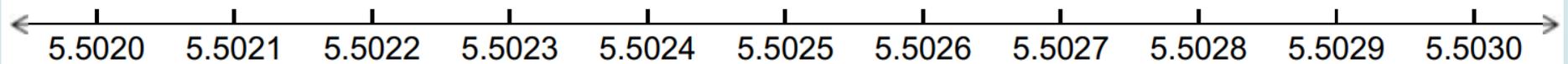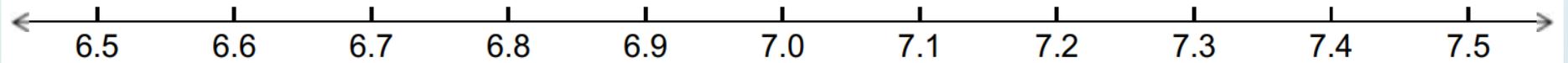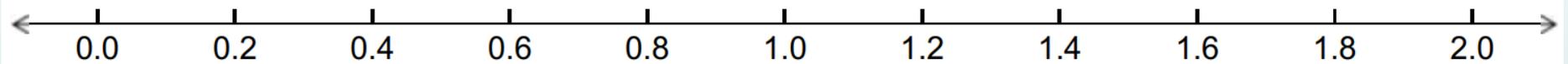
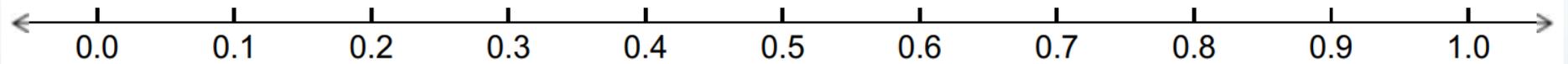# Represent ℚ on Number Lines

➡ Recall number lines:

  ➡ Whole numbers equally spaced.

  ➡ Arrows at both ends.

  ➡ Positive numbers to the right, getting greater.

  ➡ Negative numbers to the left, getting lesser.

Represent ℚ on Number Lines

➡ A number line can be broken down further to show decimals between numbers:

0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0

6.5	6.6	6.7	6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.5

5.5020	5.5021	5.5022	5.5023	5.5024	5.5025	5.5026	5.5027	5.5028	5.5029	5.5030

Represent ℚ on Number Lines

➡ Draw a suitable number line which shows each of the following numbers and at least three numbers on each side of it:

a) 3.3

b) 6.5

c) 1111

d) 21.7

e) 208.36

f) 1 000 001

g) 8765.4321

Perform Arithmetic in ℚ

➥ Adding, subtracting, multiplying, and dividing decimals works the same way as for integers – we just need to keep the location of the decimal point in mind.

➥ To add or subtract, line up the numbers so their place values match.

➥ e.g. 5.3 + 7.28   e.g. 12.85 − 5.2

$$
\begin{array}{r}
5.30 \\
+\quad 7.28 \\
\hline
12.58
\end{array}
\qquad
\begin{array}{r}
12.85 \\
-\quad 5.20 \\
\hline
7.65
\end{array}
$$

You may have to add zeroes to the start or end of a decimal so their place values match

Perform Arithmetic in $\mathbb{Q}$

➡ To multiply, your answer has as many decimal places as the total number of decimal places in your multipliers.

➡ e.g. 5.3×7.2 e.g. 3.85×2.4

	5.3
×	7.2
	6
	100
	210
	3500
	38.16

	3.85
×	2.4
	20
	320
	1200
	100
	1600
	6000
	9.240

There is no need to add zeroes when multiplying, but you can if you like.

Perform Arithmetic in ℚ

- To divide, your answer has as many decimal places as the difference between the decimal places in your dividers.

- e.g. $6 \div 1.2$

$$
\begin{array}{r}
0\ 5 \\
\hline
1.2\ \big|\ 6\ .0 \\
-\ \underline{0} \\
6\ 0
\end{array}
$$

e.g. $1.7 \div 0.2$

$$
\begin{array}{r}
0\ 8\ .5 \\
\hline
0.2\ \big|\ 1\ .7\ \ 0 \\
-\ \underline{0} \\
1\ 7 \\
-\ \underline{1\ 6} \\
1\ \ \ 0
\end{array}
$$

Perform Arithmetic in ℚ

➡ Calculate each of the following:

a) $4.9 + 1.7$

b) $7.2 + 7.6$

c) $9.47 + 6.48$

d) $1.16 + 8.93$

e) $31.63 + 42.87$

f) $8.5 - 6.4$

g) $6.3 - 5.8$

h) $9.43 - 5.41$

i) $6.58 - 5.65$

j) $41.72 - 25.83$

k) 5.9×5.8

l) 6.5×8.5

m) 6.96×1.14

n) 6.62×3.73

o) 23.37×70.77

p) $4.2 \div 2.1$

q) $9.1 \div 1.3$

r) $42.6 \div 7.1$

s) $6.82 \div 6.2$

t) $7.344 \div 6.12$

Convert from Fractions to Decimals

- Since fractions represent division, it is straightforward to convert from fractions to decimals by dividing.

- e.g. $\frac{2}{8} = 2 \div 8 = 0.25$

- Convert each of the following fractions to decimals:

a) $\frac{6}{5}$

b) $\frac{9}{12}$

c) $\frac{32}{20}$

d) $\frac{19}{25}$
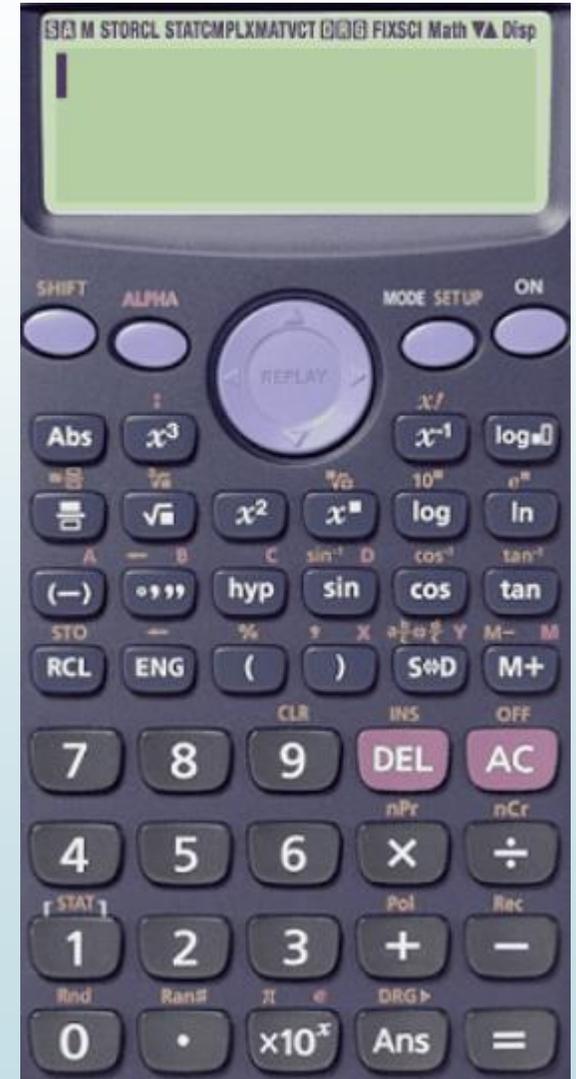
e) $\frac{13}{10}$

f) $\frac{16}{5}$

g) $\frac{7}{2}$

h) $\frac{983}{100}$

i) $\frac{110}{25}$

j) $\frac{463}{50}$

Convert from Fractions to Decimals

➡ Your calculator can do this conversion for you.

1. Type the fraction in as usual or using the division sign.

2. Press the "=" button.

3. Press the "S⇔D" button.

Convert from Decimals to Fractions

- As decimal places represent fractions of powers of ten, we can write a simple fraction with the largest necessary power of ten, then simplify if required.

- e.g. $5.34 = \dfrac{534}{100} = \dfrac{267}{50}$

- Convert each of the following decimals to fractions:

a) 0.9

b) 0.83

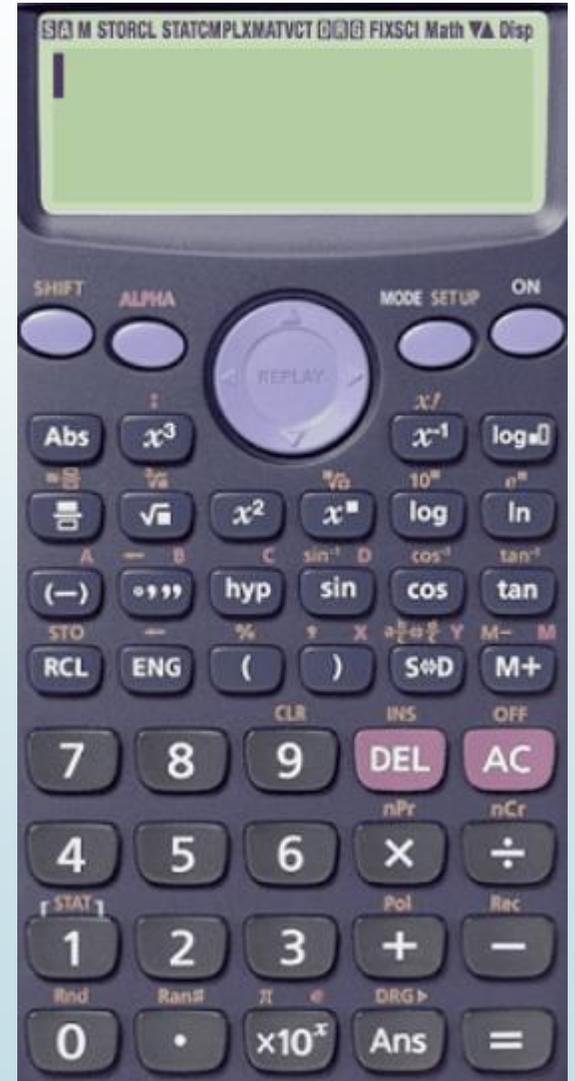c) 5.06

d) 6.78

e) 0.48

f) 0.44

g) 5.25

h) 10.55

i) 6.02

j) 7.8

Convert from Decimals to Fractions

➡ Your calculator can do this conversion for you.

1. Type the decimal in to your calculator as normal.

2. Press the "=" button.

3. Press the "S⇔D" button if necessary.

Round a Number

- **Rounding** a number is when you approximate a number to one with fewer digits.

- Rounding is a decision about what is the closest approximation you can make.

- e.g. copy the number line below and plot 5.135 as accurately as you can.

```
5.1    5.11   5.12   5.13   5.14   5.15   5.16   5.17   5.18   5.19   5.2
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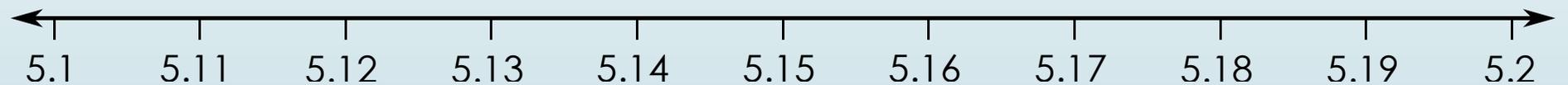
- Is 5.135 closer to 5.1 or closer to 5.2 on the number line?

Round a Number

➡ You may be asked to round to a particular place value (e.g. to nearest hundred or to two decimal places) or to a number of significant figures.

➡ Decimal places start counting from the tenths position.

➡ Significant figures start counting from the first non-zero digit.

d.p. = decimal place
s.f. = significant figure

first s.f.
↓
1234.56789
↑
first d.p.

first s.f.
↓
0.0004567
↑
first d.p.

Round a Number

➼ Significant figures stop counting at the last non-zero digit if it is before the decimal point or the last digit if it is after the decimal point.

last s.f.
↓
12300

last s.f.
↓
1.2300

Round a Number

- To round without a number line, look at the digit after the rounding digit.

- If it is between 0 and 4, round left / down.

- If it is between 5 and 9, round right / up.

- e.g. round 5.135 to one decimal place.

- Look at the second decimal place: 5.135

- It is between 0 and 4, so we round left / down: 5.1

Round a Number

➥ e.g. round 51350 to the nearest hundred.

➥ Look at the tens place: 51350

➥ It is between 5 and 9, so round right / up: 51400

➥ e.g. round 123.456 to four significant figures.

➥ Look at the fifth significant figure: 123.456

➥ It is between 5 and 9, so round right / up: 123.5

Round a Number

➥ Round each of the following to the specified accuracy:

a) 62 to the nearest ten.

b) 671 101 to the nearest hundred thousand.

c) 9 289 459 to the nearest hundred.

d) 1.174 to two decimal places.

e) 1.083 to one decimal place.

f)   1.2837 to three decimal places.

g) 89 768 to three significant figures.

h) 6 438.846 to four significant figures.

i)   396 to two significant figures.