1

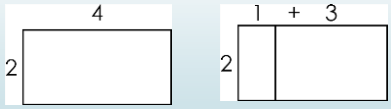
Factors – Learning Outcomes

- Factorise expressions such as:
- ax, axy, where $a \in \mathbb{Z}$
- $abxy + ay$, where $a, b \in \mathbb{Z}$
- $sx - ty + tx - sy$, where x, t, x, y are variable
- $ax^2 + bx$, where $a, b \in \mathbb{Z}$
- $x^2 + bx + c$, where $b, c \in \mathbb{Z}$
- $x^2 - a^2$

2

Factorise ax, axy

- Recall how to distribute:
- Distribution allows operations to break BEMDAS without changing the answer.
- e.g. $2 \times (1 + 3) = 2 \times 1 + 2 \times 3$



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Factorise ax, axy

- Write down what a factor is.
- A factor is a number that divides evenly ("goes into") another number.

1. Write down the factors of 12 and 20, and find the highest common factor.
2. Write down the factors of 8 and $8x$, and find the highest common factor.
3. Write down the factors of $6x$ and 8, then find the highest common factor.

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Factorise ax, axy

a) Find the highest common factor of 12 and $15x$.

b) What is $\frac{12}{3}$?

c) What is $\frac{15x}{3}$?

d) To factorise $12 + 15x$, we write $3(4 + 5x)$

e) What do you get if you distribute $3(4 + 5x)$?

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Factorise $axby + ay$

a) Find the highest common factor of 16 and $12x$.

b) What is $\frac{16}{4}$?

c) What is $\frac{12x}{4}$?

d) Factorise $16 + 12x$.

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Factorise $axby + ay$

a) Find the highest common factor of ax and bx.

b) What is $\frac{ax}{x}$?

c) What is $\frac{bx}{x}$?

d) Factorise $ax + bx$.

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Factorise $axby + ay$

- Factorise each of the following:

a) $14x + 7$
b) $4x + 20$
c) $6x + 9y$
d) $5x + 15y$
e) $3x - 9y$
f) $2x + 12xy$
g) $4xy + 32yz$
h) $3x + 6y + 9z$

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Factorise $axby + ay$

2003 OL P1 Q5
- Factorise $2xy - 4xw$

2005 OL P1 Q5
- Factorise $4ab + 8b$

2006 OL P1 Q5
- Factorise $xy + wy$
- Factorise $4a^2 + 8a$

2015 OL P1 Q9
- Factorise fully $7x - 21y$

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Factorise $sx - ty + tx - sy$

This is called factorising by grouping.

- Take $rt - 4t + 2r - 8$ as an example.
- There is no common factor for all four terms.

a) Factorise $rt - 4t$.
b) Factorise $2r - 8$.

- Write the expression as the sum of the answers to a) and b).
- $t(r - 4) + 2(r - 4)$
- Note that $(r - 4)$ is now a common factor. Factorising again gives:
- $(r - 4)(t + 2)$

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Factorise $sx - ty + tx - sy$

- Factorise $6 + 3x + cx + 2c$

a) Factorise $6 + 3x$

b) Factorise $cx + 2c$

- Write the expression as the sum of a) and b).
- Note that $3(2 + x) + c(x + 2)$ both have $(x + 2)$. ($x + 2$ and $2 + x$ are the same thing mathematically).
- Factorising gives $(x + 2)(3 + c)$

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Factorise $sx - ty + tx - sy$

- Factorise each of the following:

a) $cd - 3c - ad + 3a$

b) $xc + xb + yc + yb$

c) $10xy + 14x + 15y + 21$

d) $10xz + 65x + 8yz - 52y$

e) $20xy + 9 + 12x + 15y$

f) $p(x + y) - x - y$

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Factorise $sx - ty + tx - sy$

2003 OL P1 Q5

2004 OL P1 Q5

2005 OL P1 Q5

2006 OL P1 Q5

2016 OL P1 Q8

- Factorise $ab - 2ac + 3b - 6c$
- Factorise $3x - 3y + ax - ay$
- Factorise $ab + 2ac + 5b + 10c$
- Factorise $ax - ay + bx - by$
- Factorise $3ax + ay + 3cx + cy$

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Factorise $ax^2 + bx$

- e.g. Factorise $2x^2 + 5x$
- What is the highest common factor?
- $\frac{2x^2}{x} = 2x$
- $\frac{5x}{x} = 5$
- Factorising gives:
- $2x^2 + 5x = x(2x + 5)$

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Factorise $ax^2 + bx$

- e.g. Factorise $6x^2 + 8x$
- What is the highest common factor?
- $\frac{6x^2}{2x} = 3x$
- $\frac{8x}{2x} = 4$
- Factorising gives:
- $6x^2 + 8x = 2x(3x + 4)$

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Factorise $ax^2 + bx$

- Factorise each of the following:

a) $3x^2 + 7x$
b) $8x^2 - 4x$
c) $6x^2 + 24x$
d) $6x^2 + 10x$
e) $4y^2 + 3y$
f) $m^4 - 3m^2$
g) $6x^2 + 8x + 12yx$

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Factorise $x^2 + bx + c$

These expressions are called quadratic trinomials.

- We will cover two methods for factorising quadratic trinomials.
- Firstly, guide number.
- e.g. Factorise $x^2 + 7x + 12$

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Factorise $x^2 + bx + c$

factor pairs of c are sets of two numbers which multiply to get c

- Identify c (the guide number) and set it aside.
- List out factor pairs of c.

$x^2 + 7x + 12$ → Aside: 12

1, 12
2, 6
3, 4
-1, -12
-2, -6
-3, -4

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Factorise $x^2 + bx + c$

- Add up the numbers in each factor pair.
- Note which pair add up to b.

$x^2 + 7x + 12$ → Aside: 12

1 + 12 = 13
2 + 6 = 8
3 + 4 = 7
-1 + -12 = -13
-2 + -6 = -8
-3 + -4 = -7

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Factorise $x^2 + bx + c$

- Replace b with the two factors you picked.
- Factorise by grouping.

$x^2 + 7x + 12$

$= x^2 + 3x + 4x + 12$

$= x(x+3) + 4(x+3)$

$= (x+3)(x+4)$

Aside: 12

$1 + 12 = 13$
$2 + 6 = 8$
$3 + 4 = 7$
$-1 + -12 = -13$
$-2 + -6 = -8$
$-3 + -4 = -7$

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Factorise $x^2 + bx + c$

- Secondly, big X.
- e.g. Factorise $x^2 + 7x + 12$
- Draw a big X.

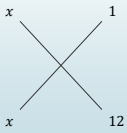
21

Factorise $x^2 + bx + c$

factor pairs of c are sets of two numbers which multiply to get c

- On each of the left ends, put an x.
- On the right ends, put a factor pair of c.

e.g. Factorise $x^2 + 7x + 12$



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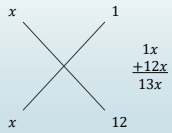
Factorise $x^2 + bx + c$

- Multiply down each line of the X.
- Add the results.

e.g. Factorise $x^2 + 7x + 12$



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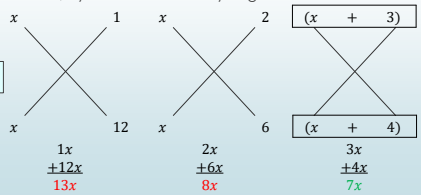
Factorise $x^2 + bx + c$

- If the sum matches b, you've found your factors.
- If not, try other factors until you get b.

e.g. Factorise $x^2 + 7x + 12$



- So $x^2 + 7x + 12 = (x + 3)(x + 4)$

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Factorise $x^2 + bx + c$

- Factorise each of the following:

a) $x^2 + 10x + 24$
b) $a^2 + 12a + 27$
c) $y^2 + 8y + 15$
d) $b^2 + 6b + 8$
e) $c^2 + 9c + 14$
f) $d^2 + 13d + 36$
g) $z^2 + 5z + 6$
h) $z^2 + 7z + 6$
i) $10e + 25 + e^2$

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Factorise $x^2 + bx + c$

- Be careful if some signs are negative:
- e.g. Factorise $x^2 - x - 12$
- $= x^2 + 3x - 4x - 12$
- $= x(x + 3) - 4(x + 3)$
- $= (x + 3)(x - 4)$

Guide number / grouping

Big X

$(x \quad + \quad 3)$

$(x \quad - \quad 4)$

$3x$
$-4x$
$-x$

Aside	-12
$1 + -12$	-11
$2 + -6$	-4
$3 + -4$	-1
$4 + -3$	1
$6 + -2$	4
$12 + -1$	11

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Factorise $x^2 + bx + c$

- Factorise each of the following:

a) $x^2 - 2x - 15$
b) $a^2 + 2a - 3$
c) $y^2 - 6y + 8$
d) $b^2 - 9b + 20$
e) $c^2 + c - 6$
f) $z^2 + 3z - 18$
g) $z^2 - 3z - 18$
h) $z^2 - 9z + 18$
i) $42 - 13d + d^2$

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Factorise $x^2 + bx + c$

2003 OL P1 Q5
- Factorise $x^2 + 2x - 8$

2005 OL P1 Q5
- Factorise $x^2 + 2x - 15$

2007 OL P1 Q5
- Factorise $x^2 - x - 90$

2008 OL P1 Q5
- Factorise $x^2 - 2x - 24$

2017 OL P1 Q11
- Factorise $x^2 + 4x - 5$. One of the factors is $(x + 5)$.

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Factorise $x^2 - a^2$

- When an expression can be written as a square minus a square, (i.e. $x^2 - y^2$) it is called a difference of two squares.
- The factorisation can be written simply $(x - y)(x + y)$.
- e.g. $x^2 - a^2 = (x - a)(x + a)$
- e.g. $m^2 - n^2 = (m - n)(m + n)$
- e.g. $7^2 - 5^2 = (7 - 5)(7 + 5) = (2)(12) = 24$
- If the two terms are not squares, they must be turned into squares first:
- e.g. $x^2 - 16 = x^2 - 4^2 = (x - 4)(x + 4)$

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Factorise $x^2 - a^2$

- Factorise each of the following:

a) $x^2 - 4$
b) $y^2 - 9$
c) $z^2 - 25$
d) $a^2 - 121$
e) $64 - b^2$
f) $10^2 - 3^2$
g) $4.2^2 - 3.8^2$

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Factorise $x^2 - a^2$

2003 OL P1 Q5
- Factorise $36 - y^2$

2004 OL P1 Q5
- Factorise $x^2 - 25$

2005 OL P1 Q5
- Factorise $x^2 - y^2$

2006 OL P1 Q5
- Factorise $p^2 - 36$

2015 OL P1 Q9
- Factorise $x^2 - 25$